

Prop 1

Assume that $i \in \{1, 2, \dots, n\}$ s.t $M_{i,j}$ is finite for all $j \in \{1, 2, \dots, n\}$

Let V_i denote the subspace of the algebra $(B(V)^{op} \# H_i^{op})^{cop}$ generated by

$$\{(\text{ad}_\sigma x_i)^{m_{ij}}(x_j) \mid j \neq i\} \cup \{y_i^R\}$$

The subalgebra B_i of $(B(V)^{op} \# H_i^{op})^{cop}$ generated by V_i is iso to $B(V_i)$

$$\text{and } \delta^+(B_i) = \{s_i \mid \delta^+(B(V)) \setminus \{s_i\}\} \cup \{-e_i\}$$

Remark 1:

$\delta^+(B_i)$ coincides with δ with respect to the basis $\{s_i \mid e_j \mid 1 \leq j \leq n\}$ in \mathbb{Z}^n

$$\delta^+(B_i) = S_i(\delta(B(V)))$$

Remark 2:

\mathbb{Z}^n Cartan type,

V_i

this transformation doesn't change Cartan type.

$$\textcircled{1} B_i = \ker y_i^L \otimes \mathbb{k}[y_i^R]$$

$\ker y_i^L$ is generated as an algebra

by $\bigcup_{j \neq i} M_{i,j}$

$$B(V_i) = \ker y_i^L \otimes \mathbb{k}[x_i]$$

$\textcircled{2} B_i$ is Nichols.

$$\textcircled{3} \delta^+(B_i) = (S_i(\delta^+(B(V))) \setminus \{e_i\}) \cup \{-e_i\}$$

Proof:

In the \mathbb{Z}^n -graded algebra

$$(B(V)^{op} \# H_i^{op})^{cop}$$

the elements x_j and y_i^R has degree

$$e_j \text{ and } -e_i \quad \text{deg}(y_i^R) = 0_{\mathbb{Z}^n}$$

Hence $(\text{ad}_\sigma x_i)^{m_{ij}}(x_j)$

$$\text{have degree } [e_j + m_{ij}e_i = S_i(-e_j)]$$

Fix the \mathbb{Z}^n -degrees of the generators of B_i

$$\text{by } \text{deg } y_i^R := -e_i / \text{deg}(\text{ad}_\sigma x_i)^{m_{ij}}(x_j) := e_j$$

Thus

$S_i(\delta^+(B_i))$ is the set of degrees of the PBW generators of B_i in $(B(V)^{op} \# H_i^{op})^{cop}$

then

$$\delta^+(\ker y_i^L \otimes \mathbb{k}[y_i^R]) \supseteq \delta^+(B(V)) \setminus \{e_i\} \cup \{-e_i\}$$

$$S_i(\delta^+(B_i))$$

$$\delta^+(B_i) \supseteq \text{ad}_\sigma(x_i)^{m_{ij}} x_j$$

$$\delta^+(B(V)) \supseteq S_i(-e_j)$$

$$\text{ad}_\sigma(x_i)^{m_{ij}} x_j$$